

Higher 2022 Paper 2

$$1. a) m_{AB} = \frac{-4 - (-1)}{2 - (-1)} = \frac{-3}{3} = -1$$

$$m_{\perp} = 1$$

$$y - 3 = 1(x - 7)$$

$$y - 3 = x - 7$$

$$y - x = -4$$

$$b) \text{mid}_{AC} = \left(\frac{-1+7}{2}, \frac{-1+3}{2} \right) = (3, 1)$$

$$m = \frac{-4 - 1}{2 - 3} = \frac{-5}{-1} = 5$$

$$y + 4 = 5(x - 2)$$

$$y + 4 = 5x - 10$$

$$y - 5x = -14$$

$$c) y - x = -4 \quad (1)$$

$$y - 5x = -14 \quad (2)$$

$$(1) - (2)$$

$$4x = 10$$

$$x = 2.5$$

$$y - 2.5 = -4$$

$$y = -4 + 2.5$$

$$y = -1.5$$

$$(2.5, -1.5)$$

$$2. b^2 - 4ac > 0$$

$$(-8)^2 - 4 \times 2 \times (4 - p) > 0$$

$$64 - 8(4 - p) > 0$$

$$64 - 32 + 8p > 0$$

$$32 + 8p > 0$$

$$8p > -32$$

$$p > -4$$

$$3a) k \sin(x + a) = k \sin x \cos a + k \cos x \sin a$$

$$4 \sin x + 5 \cos x$$

$$k \cos a = 4$$

$$k \sin a = 5$$

$$\tan a = \frac{5}{4}$$

$$a = 51$$

$$k = \sqrt{4^2 + 5^2}$$

$$= \sqrt{41}$$

$$\sqrt{41} \sin(x + 51)$$

$$\sqrt{41} \sin(x + 0.89)$$

$$\frac{51 \times \pi}{180} = 0.89$$

$$b) \sqrt{41} \sin(x+51) = 5.5$$

$$\sin(x+51) = 0.858\dots$$

$$x+51 = 59^\circ, 121, 419$$

$$x = 8^\circ, 70^\circ, 368$$

$$x = 0.14, 1.22$$

$$\frac{180}{S/A}$$

$$\frac{T/C}{T/C}$$

$$180 - 59 =$$

$$\frac{8 \times \pi}{180} =$$

$$\frac{70 \times \pi}{180} =$$

$$4. a) \int_{-1}^2 x^3 - 5x^2 + 2x + 8 \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{2x^2}{2} + 8x \right]_{-1}^2$$

$$= \left(\frac{2^4}{4} - \frac{5(2)^3}{3} + 2^2 + 8(2) \right) - \left(\frac{(-1)^4}{4} - \frac{5(-1)^3}{3} + (-1)^2 + 8(-1) \right)$$

$$= \frac{32}{3} - \frac{-61}{12}$$

$$= \frac{63}{4} \text{ units}^2$$

$$b) \int_2^4 \dots \, dx$$

$$= \left(\frac{4^4}{4} - \frac{5(4)^3}{3} + 4^2 + 8(4) \right) - \frac{32}{3}$$

$$= \frac{16}{3} - \frac{32}{3} = -\frac{16}{3}$$

$$\text{Total area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \text{ units}^2$$

$$5a) i) (3x+5)^2 - 2 = 9x^2 + 30x + 25 - 2 = 9x^2 + 30x + 23$$

$$ii) 3(x^2 - 2) + 5 = 3x^2 - 6 + 5 = 3x^2 - 1$$

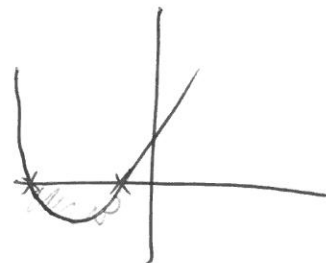
$$b) 9x^2 + 30x + 23 < 3x^2 - 1$$

$$6x^2 + 30x + 24 < 0$$

$$6(x^2 + 5x + 4) < 0$$

$$(x+4)(x+1)$$

$$x = -4 \quad x = -1$$



$$-4 < x < -1$$

$$\begin{aligned}
 6. \quad y &= \int 1 - \frac{3}{x^2} dx = \int 1 - 3x^{-2} dx \\
 &= x - \frac{3x^{-1}}{-1} + c \\
 &= x + 3x^{-1} + c \\
 y &= x + \frac{3}{x} + c
 \end{aligned}$$

sub $\begin{pmatrix} 3, 6 \\ x, y \end{pmatrix}$

$$6 = 3 + \frac{3}{3} + c$$

$$6 = 3 + 1 + c$$

$$c = 2$$

$$\underline{\underline{y = x + \frac{3}{x} + 2}}$$

$$7. \quad y = kx^n$$

$$\begin{aligned}
 \log_5 y &= \log_5 kx^n \\
 &= \log_5 k + \log_5 x^n
 \end{aligned}$$

$$\log_5 y = \log_5 k + n \log_5 x$$

$$y = mx + c$$

$n = \text{gradient}$

$\log_5 k = y \text{ intercept}$

$$n = \frac{3 - 1}{0 - 2} = \frac{4}{-2} = -2$$

$$\log_5 k = 3$$

$$5^3 = k$$

$$k = 125$$

$$\underline{\underline{y = 125x^{-2}}}$$

$$8. a) \quad xy = 150$$

$$y = \frac{150}{x}$$

$$A = (x-3)(y-2)$$

$$A = xy - 2x - 3y + 6$$

$$= x\left(\frac{150}{x}\right) - 2x - 3\left(\frac{150}{x}\right) + 6$$

$$= 150 - 2x - \frac{450}{x} + 6$$

$$= 156 - 2x - \frac{450}{x} \quad \text{as required}$$

$$b) A(x) = 156 - 2x - 450x^{-1}$$

$$A'(x) = -2 + 450x^{-2} = 0$$

$$-2 + \frac{450}{x^2} = 0$$

$$\frac{450}{x^2} = 2$$

$$x^2 = 225$$

$$x = \pm 15$$

$$x = 15$$

$$A = 156 - 2(15) - \frac{450}{15}$$

$$= \underline{\underline{96 \text{ m}^2}}$$

x	$\xrightarrow{10}$	15	$\xrightarrow{20}$
A'	$+$	0	$-$
slope	$/$	$-$	\backslash

$$9a) x^2 + (3x+7)^2 - 4x - 6(3x+7) - 7 = 0$$

$$x^2 + 9x^2 + 42x + 49 - 4x - 18x - 42 - 7 = 0$$

$$10x^2 + 20x = 0$$

$$10x(x+2) = 0$$

$$x=0 \quad x=-2$$

$$y = 3(0) + 7 \quad y = 3(-2) + 7$$

$$= 7 \quad = 1$$

$$(0, 7) \quad (-2, 1)$$

$$b) \text{mid}_{PA} = \left(\frac{0-2}{2}, \frac{7+1}{2} \right) = (-1, 4)$$

$$\text{centre} : (2, 3)$$

$$\text{radius} \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\underline{\underline{(x-2)^2 + (y-3)^2 = 10}}$$

$$10. a) P = 4.99087 \left(\frac{\cancel{24.55}}{42.5 - 24.55} \right)^{1.81}$$

$$= 929 \text{ points}$$

$$b) 850 = 0.188807 (600 - 210)^k$$

$$850 = 0.188807 \times 390^k$$

$$4502 = 390^k$$

$$\implies \log 4502 = \log 390^k$$

$$\log 4502 = k \log 390$$

$$\log_{390} 4502 = k$$

$$k = 1.41$$

$$k = \frac{\log 4502}{\log 390} = \underline{1.41}$$